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A Constitutive Framework for the Numerical Analysis of Organic Soils and Directionally Dependent Materials

FracMan Technology Group

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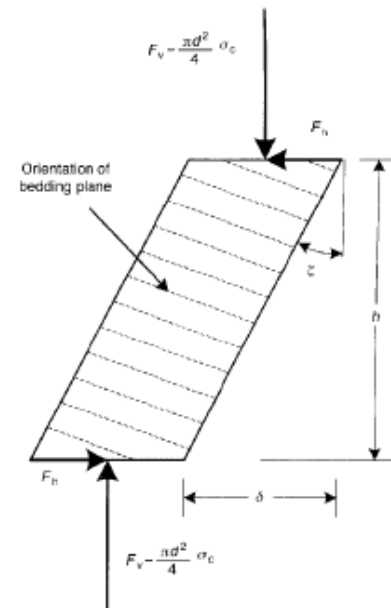
Presentation Outline

- Some Physical Observations of Organics;
 - Fibrous nature;
 - Design Requirements; and
 - Laboratory Testing.
- Numerical Modelling
 - Its basic origins;
 - Material types
- A Generalised Numerical Constitutive Framework;
 - Applicability;
 - Basic mechanical strength envelope;
 - Creep and Rate Dependency;
 - Inclusion of Orthotropy; and
 - Porous Flow & Thermal dependency.



Some Features of Organic Soils

- Mixture of fragmented organic material formed in wetlands ;
- The soil has essentially an open structure with interstices filled with a secondary structural arrangement of nonwoody, fine fibrous material. If >20% fibre content classified as fibrous;
- Fibrous peat differs from amorphous peat in that it has a low degree of decomposition and easily recognizable structure;
- The compressibility of fibrous peat is very high and so it's rate of consolidation;
- Formation of peat deposits leads to a pronounced structural anisotropy in which the fibres tend to have horizontal orientation; and
- Under a consolidation, deformation is directionally dependent and water tends to flow faster from the soil in the horizontal direction than in the vertical direction.





Design Requirements

- When designing a foundation for an embankment its influence on its surroundings is often an important issue;
- In considering this influence a proper indication of horizontal deformations and horizontal stresses in the subsoil is needed;
- Many sediment deposits are deposited in horizontal strata, so it is to be expected that their mechanical properties in both horizontal directions might differ from their properties in vertical direction;
- It is widely recognized that soft soil might show anisotropic behaviour;
- Soft soil is often described as an anisotropic heterogeneous material. Each soil property like permeability, stiffness or strength might show anisotropic behaviour.



Laboratory Testing

- Laboratory testing methods to test for the shear strength of peat are generally the same as for traditional soils.
- Triaxial compression (TXC), direct shear (DS), direct simple shear (DSS) and ring shear (RS) have been used to measure undrained and effective strength properties.
- Laboratory testing of peat strength properties is complicated by several factors, as follows.
 - It is difficult to obtain and prepare samples because of the high water content and fibres.
 - Corrections related to apparatus compliance and membrane stiffness can be a large percentage of the measured strength.
 - Interpretation of actual failure is difficult because of the large strains involved and excessive deformation.
 - Structural anisotropy as a result of the presence of fibres within peat can cause artificial reinforcement of the sample



Numerical Modelling

■ What is Numerical Modelling?

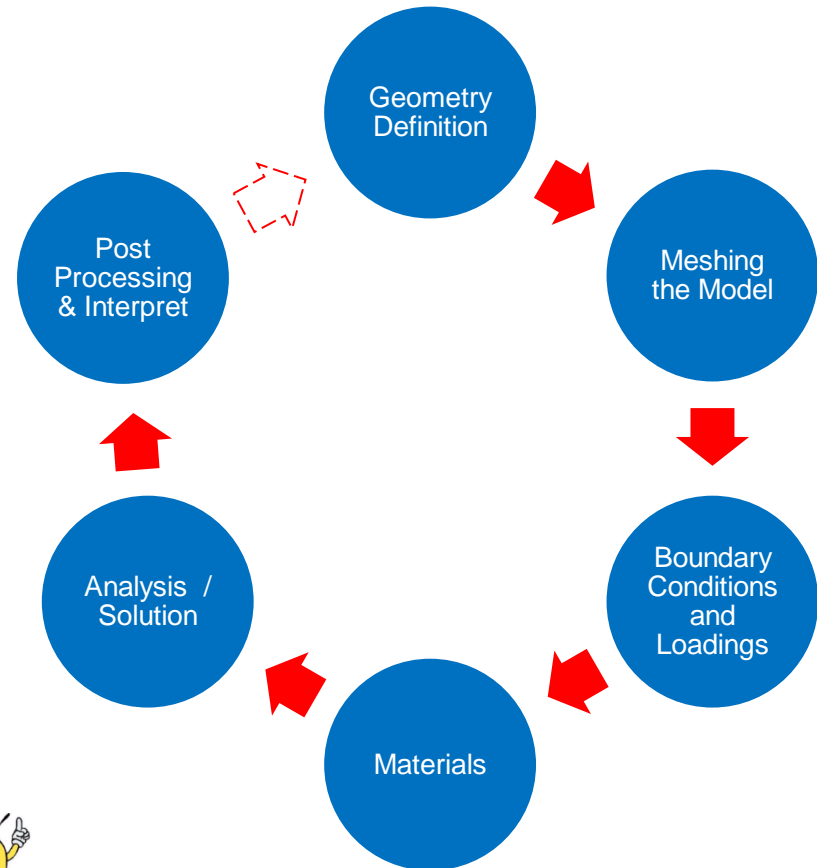
- *Wikipedia (2009), “A computer simulation, a computer model or a computational model that attempts to simulate a particular system. Computer simulations have become useful in the process of engineering, . . . to gain insight into the operation of those systems, or to observe or understand their behaviour or limits.”*

■ Types of Numerical Modelling?

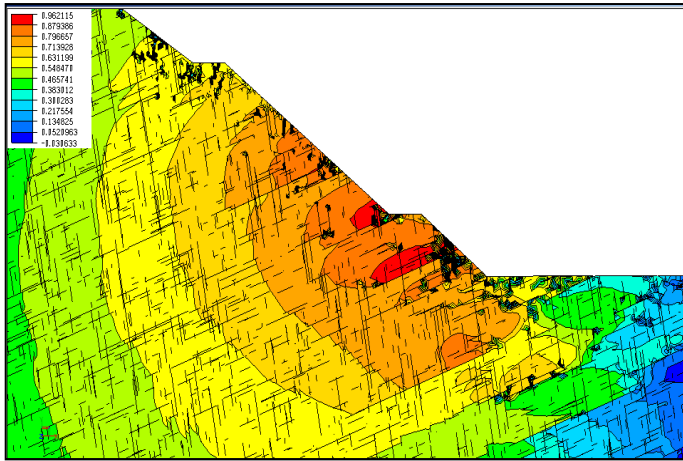
- Semi analytical approximations;
- Stress modelling – continuous/discontinuous

■ What they all have in common is .

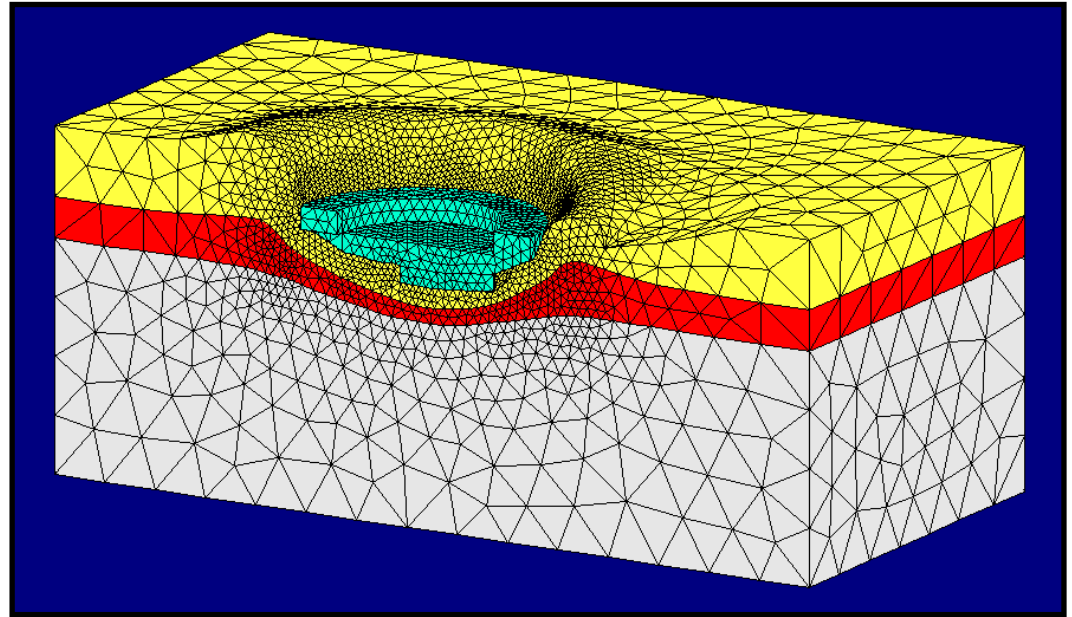
- They all look to solve Complex Equations that bound a specific problem



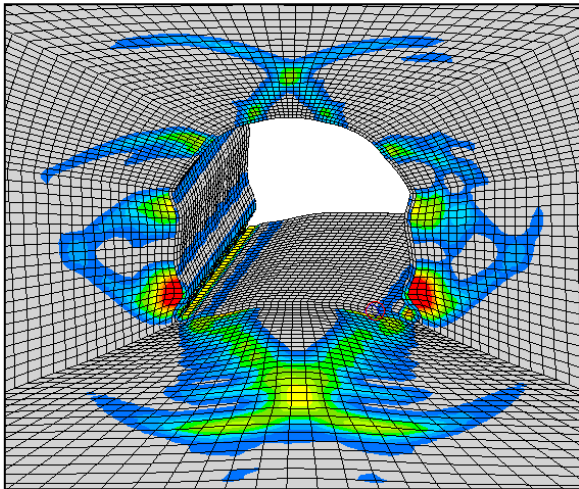
Geomechanical Modelling



Slope Stability



Soft soil foundations



Tunnel Integrity

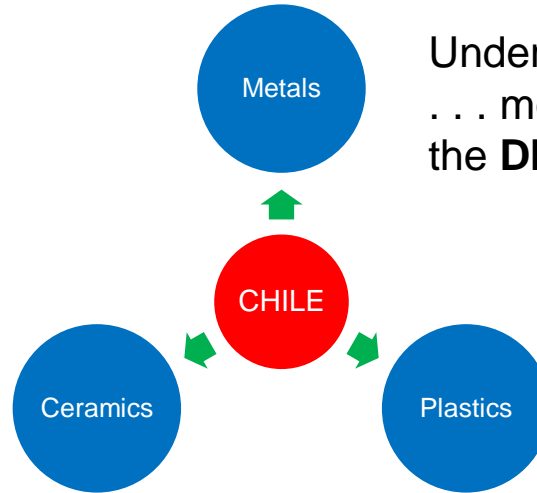


Material Characterisation

Categories of Material

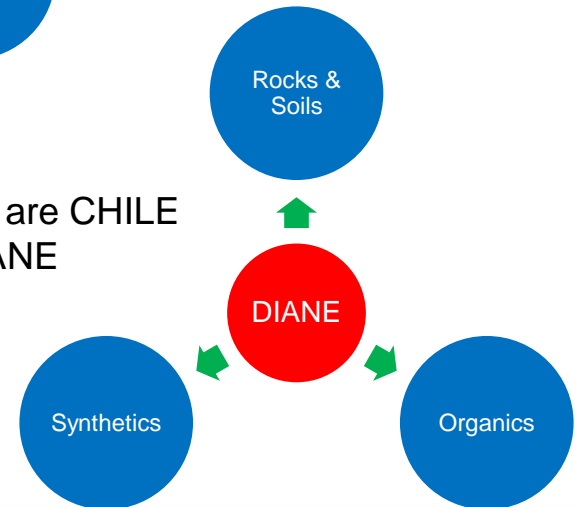
- **C**ontinuous;
- **H**omogeneous;
- **I**sotropic;
- **L**inear
- **E**lastic.

- **D**iscontinuous;
- **I**nhomogeneous;
- **A**nisotropic;
- **N**ot
- **E**lastic.



Under extreme loading conditions . . . most **CHILE** materials fall into the **DIANE** classification anyway!!

A few numerical approaches are CHILE
→ . . . but most today are DIANE





Constitutive Model Overview

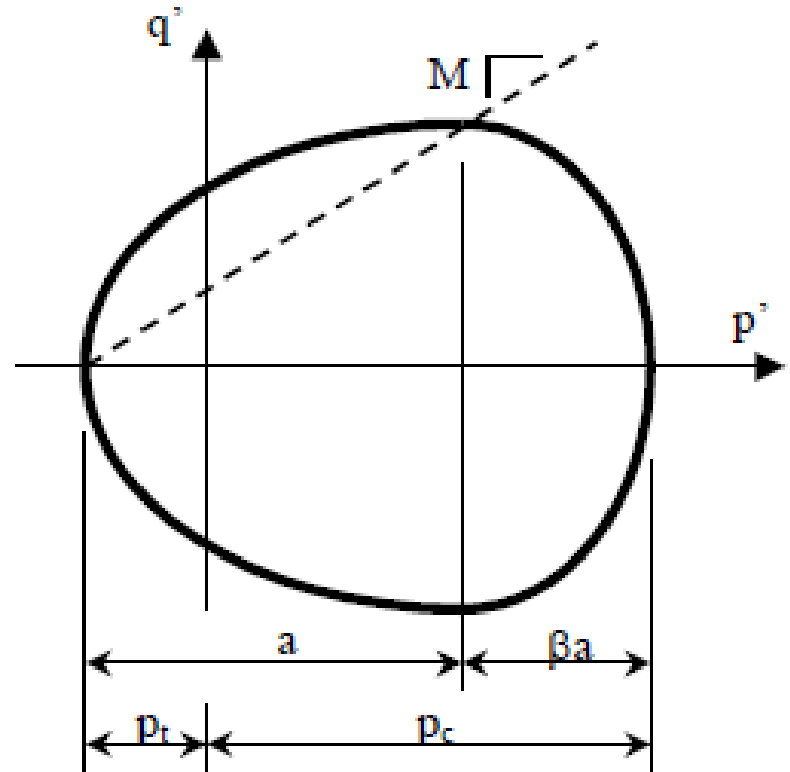
- Several models based on Critical State Theory including:
 - Elastic and Poro-Elastic models
 - Orthotropic Modified Cam Clay (transverse isotropic materials);
- A series of soft soil/rock models based on critical state theory. These have different levels of sophistication including:
 - Standard hardening/softening formulations;
 - Rate dependent hardening/softening formulations;
 - Combined short-term rate dependent failure and long-term creep;
 - Models which represent the evolution of the material state due to lithification;
 - Depth (porosity) dependent initial yield surface.
- Traditional strength criteria may also be employed e.g. Mohr Coulomb, Drucker-Prager, etc.



Isotropic Critical State Failure Model

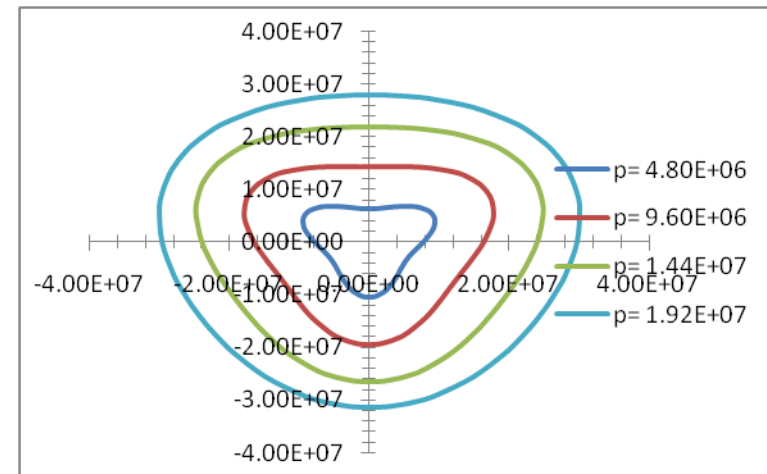
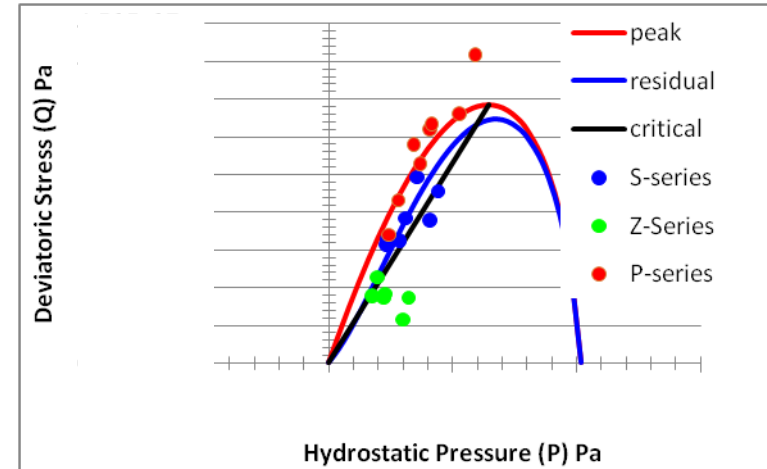
- The critical state approach is often selected since its basis is derived from the experimental measurement of soils at large strain conditions;
- The isotropic Cam Clay form has been frequently employed, but with the inclusion of a number of particular aspects to allow improved representation of many different material types, including:
 - Fibrous materials such as peat;
 - Laminated shales;
 - Time (rate/creep) dependent materials such as chalk.

- The form of the failure surface in Pressure (P) vs Deviatoric Stress (q) space;



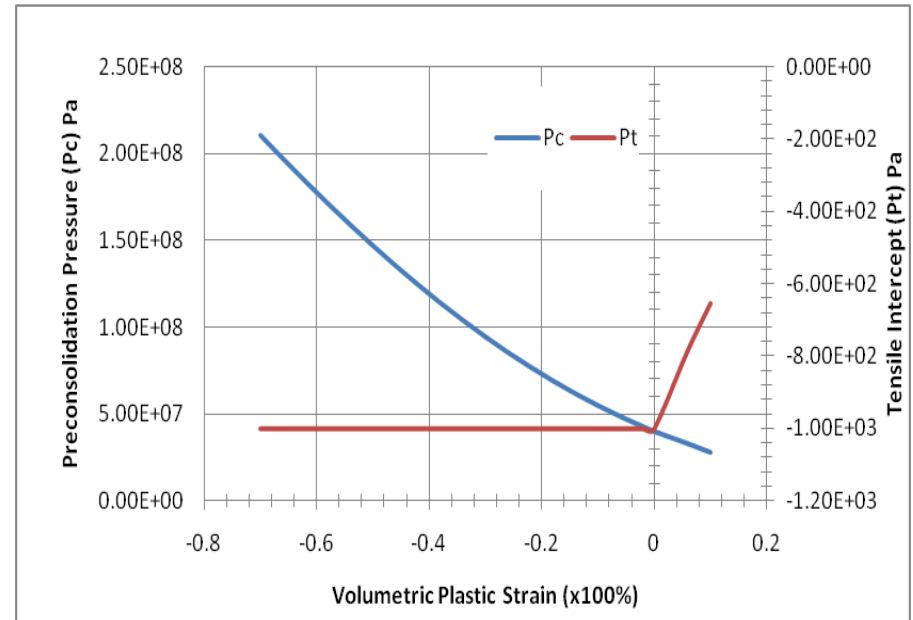
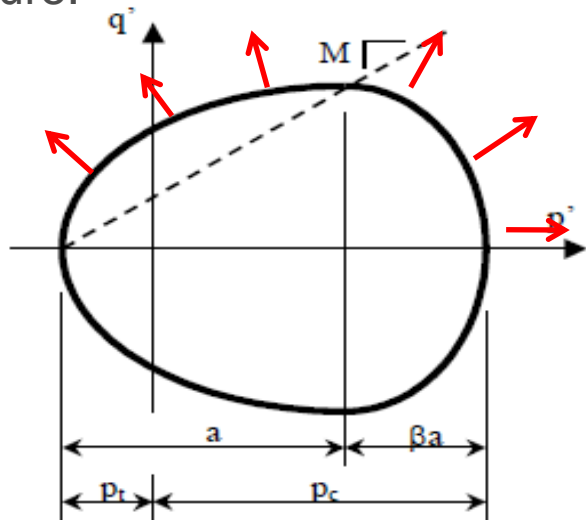
Isotropic Critical State Failure Model

- Components of the characterised constitutive framework:
 - Transverse isotropic elasticity;
 - Orthotropic pressure dependent yielding surface based on critical state soil mechanics;
 - A smoothly varying yield surface, that is non-circular in the deviatoric plane;
 - A non-associated flow rule;
 - Mesh objectivity of the solution is achieved by incorporating fracture energy concepts;



Critical State Failure Model – Strain Hardening

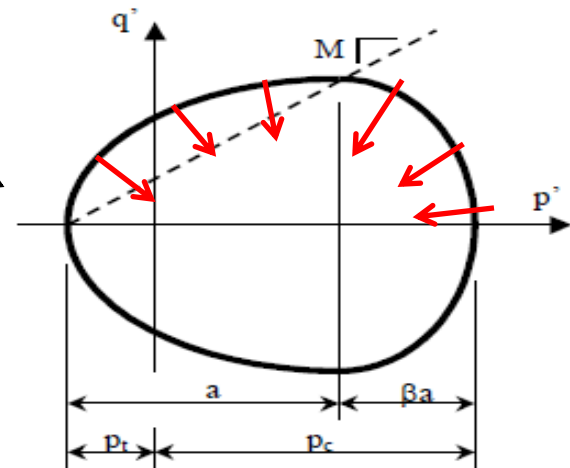
- Hardening of the model is achieved through expansion of the yielding surface (by variation of the two parameters P_c and P_t with respect to the volumetric plastic strain level).
- This then modifies the size of the yielding envelope based on material failure.



Critical State Failure Model – Creep

- The incremental creep strain is defined as
- Defines the relationship between the rate independent pre-consolidation pressure (p_c) and the pre-consolidation pressure of the rate dependent surface (p_c^*),
- The creep strain rate is therefore nonlinear dependent on both:
 - The relative magnitude of the rate independent and rate dependent surfaces.
 - The magnitude of deviatoric stress; i.e. the creep strain rate increases at higher deviatoric stress levels.

$$\Delta \epsilon_c = \Delta t \gamma_c \Gamma^{n_c - 1} \left(1 + \frac{q}{p_c} \right)^{m_c}$$



$$\Gamma = \frac{p_c^*}{p_c} = \left[1 + \frac{\Delta \epsilon_c}{\Delta t \gamma_c} \left(1 + \frac{q}{p_c} \right)^{-m_c} \right]^{1/n_c}$$



Critical State Failure Model

■ The Isotropic Yield Surface

- The form of the yielding function used to define the failure surface is given by:

$$\Phi(\sigma, \varepsilon_v^p) = \left(\frac{q}{M}\right)^2 + \frac{1}{b^2}(p - p_t + a)^2 - a^2 = 0$$

- Where $b = \begin{cases} 1 & p \geq (P_t - a) \\ \beta & p < (P_t - a) \end{cases}$

$$a = \frac{1}{1 + \beta}(p_t - p_c); \quad b = \begin{cases} 1 & p \geq (p_t - a) \\ \beta & p < (p_t - a) \end{cases}$$

- Where P is the effective mean stress, P_c is the pre-consolidation pressure, and P_t is the tensile intercept (strength).
- The term beta is a material constant that defines the shape of the consolidation side of the failure envelope.

- The deviatoric stress q is defined in the standard manner of:

$$q = \sqrt{3J_2'} = \sqrt{\frac{3}{2} \mathbf{S} : \mathbf{S}} = \sqrt{\frac{1}{2} \sigma^T \mathbf{P} \sigma}$$

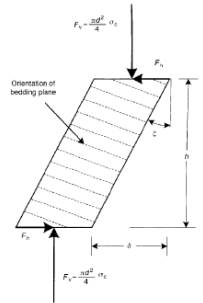
- Where J_2 is the second invariant of the deviatoric stress tensor \mathbf{S} ;
- We include the projection matrix \mathbf{P} such to compute the deviatoric stress from the globally aligned stress tensor sigma.
- We define projection matrix as the following:

$$\mathbf{P} = \begin{bmatrix} \Omega & 0 \\ 0 & \Gamma \end{bmatrix} \quad \Omega = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$



Structurally Anisotropic Materials

- Most typical features of such materials:
- The variation of the compressive strength with the angle between the “fabric/bedding” and loading is such that the maximum strength occurs when the loading direction is either P or N to the layering;
- The minimum stress occurs when the loading to “fabric/bedding” orientation ranges $30^\circ - 60^\circ$ where high shear induces failure on the laminations;
- The elastic properties are transverse isotropic with the Elastic Modulus normal to the “fabric/bedding” being less than the in plane;
- The elastic properties are a nonlinear function of confining pressure and effective stress.





Structurally Anisotropic Materials

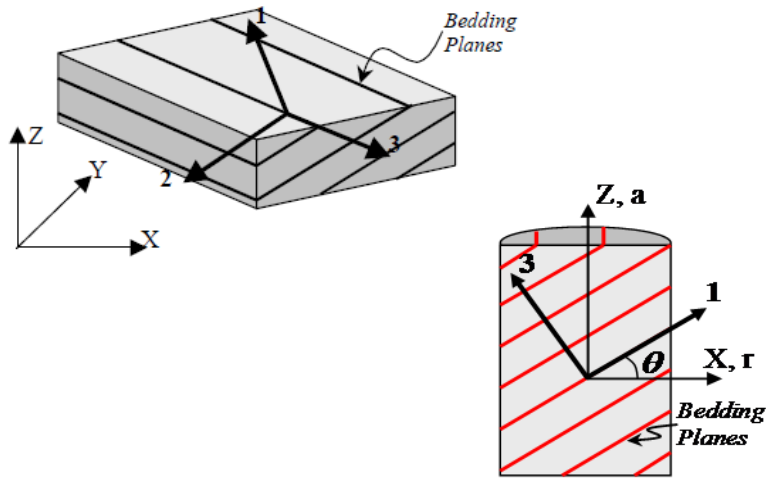
- The transverse isotropic nature of the material may be represented within a finite element framework in a number of ways, these include:
- Inclusion of an embedded weakness description via a smeared law, however this neglects the true compactive behaviour of the materials at higher confinement levels.
- Direct representation of the “fabric/bedding” weakness as an interface with adhesive, cohesive and frictional properties, however due to fabric dimensions this is just not practical.
- Representation of the macroscopically observed deformations using a phenomenological constitutive model based, for example orthotropic elastoplasticity.



Orthotropic Transformation

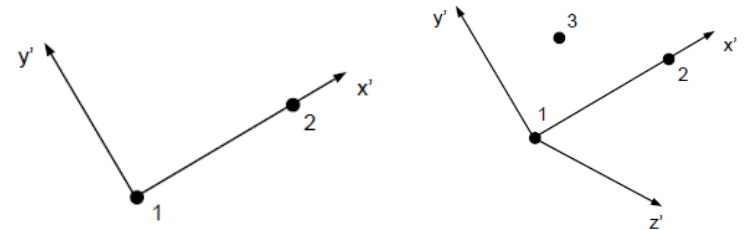
■ Orthotropic Transformation

- We employ a local coordinate system to transform global coordinate measures (material variables) to an alternative coordinate system that coincides with material fabric directions;



■ Local Coordinate System Definition

- We use a simple specification of three nodal points to define the system;
- Node 1 defines the origin of the coordinate system and node 2 defines the local x' axis;
- For three-dimensional applications, node 3 defines the x' - y' plane.





Transverse Isotropic Elasticity

- We define the elastic response of any orthotropic material through use of the locally orientated (transformed) coordinate system by using the linear Hooke's Law relationship:

$$\sigma_e^l = \mathbf{D}_{orth}^l \varepsilon_e^l$$

- Here \mathbf{D} is the matrix of the stiffness constants that interrelates the elastic stress and strain vectors respectively. The superscript l indicates the relationship is defined in the orientated material fabric coordinate system.
- For transverse isotropic materials like Opalinus Clay we only need specify nine elastic (stiffness) constants to provide an appropriate response;

- The required elastic constants are:
 - E_1 – the Elastic Modulus in the direction normal to the bedding plane
 - E_2 – the in plane Elastic Modulus;
 - G_{12} – the out of plane Shear Modulus;
 - ν_{23} – the in plane Poisson ratio; and
 - ν_{12} – the out of place Poisson ratio.
- Of these constants it is possible to estimate the shear modulus variable G_{12} from the standard St-Venants formula:

$$\frac{1}{G_{12}} = \frac{1}{E_1} + \frac{1}{E_2} + 2 \frac{\nu_{12}}{E_1}$$



Critical State Failure Model - Orthotropy

■ The Orthotropic Extension

- The isotropic yielding function may be extended to consider orthotropic behaviour by rewriting of the deviatoric stress equation, such that:

$$q_{orth} = \sqrt{\frac{1}{2}(\sigma^i)^T P_{orth} \sigma^i}$$

- Leading to the orthotropic yielding function to take the modified form of:

$$\Phi(\sigma, \varepsilon_v^p) = \left(\frac{\xi(\theta) q_{orth}}{2M} \right)^2 + \frac{1}{b^2} (p_{orth} \sigma - p_t + a)^2 - a^2 = 0$$

- Where the projection matrix **P** is now written in orthotropic notation, such that:

$$P_{orth} = \begin{bmatrix} \Omega_{orth} & 0 \\ 0 & \Gamma_{orth} \end{bmatrix}$$

- As defined in previous notation the vector of stress variables that are aligned with the fabric of the material;
- We also make use of Hill's orthotropy notation derived for metals, in that

$$\Omega_{orth} = \begin{bmatrix} 2(\alpha_4 + \alpha_6) & -2\alpha_4 & -2\alpha_6 \\ -2\alpha_4 & 2(\alpha_4 + \alpha_5) & -2\alpha_5 \\ -2\alpha_6 & -2\alpha_5 & 2(\alpha_5 + \alpha_6) \end{bmatrix}$$

$$\Gamma_{orth} = \begin{bmatrix} 6\alpha_7 & 0 & 0 \\ 0 & 6\alpha_8 & 0 \\ 0 & 0 & 6\alpha_9 \end{bmatrix}$$

$$P_{orth} = [\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad 0 \quad 0 \quad 0]$$

- Where the material constants α_1 - α_9 control the failure value in the orthotropic framework;
- For the transverse isotropic case for the Opalinus Clay this simplifies to only α_4 - α_9 being needed

Critical State Failure Model - Orthotropy

- The transverse isotropic condition then follows that:

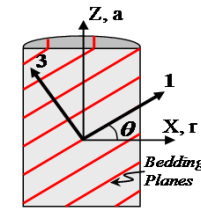
$$\alpha_4 = \alpha_6, \quad \alpha_7 = \alpha_9, \quad \alpha_8 = \frac{2}{3}(2\alpha_4 + \alpha_5)$$

- This then permits the orthotropy matrices to be simplified to:

$$\mathbf{Q}_{orth} = \begin{bmatrix} 4\alpha_4 & -2\alpha_4 & -2\alpha_4 \\ -2\alpha_4 & 2(\alpha_4 + \alpha_5) & -2\alpha_5 \\ -2\alpha_4 & -2\alpha_5 & 2(\alpha_4 + \alpha_5) \end{bmatrix}$$

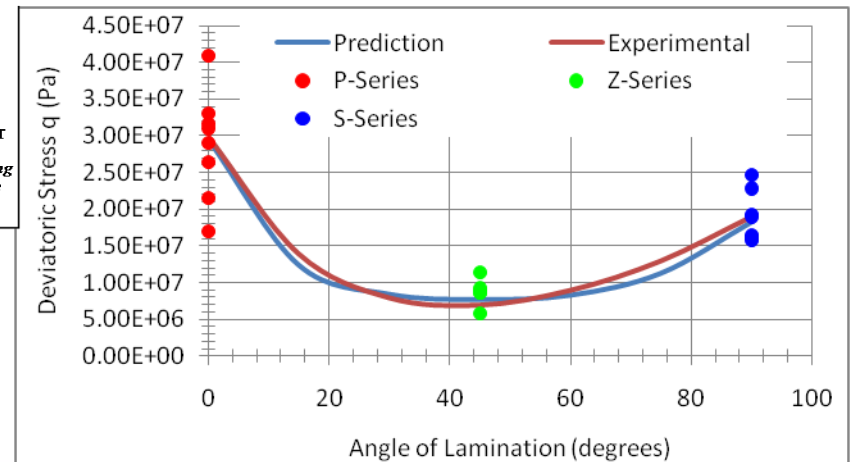
$$\mathbf{\Gamma}_{orth} = \begin{bmatrix} 6\alpha_7 & 0 & 0 \\ 0 & 4(2\alpha_4 + \alpha_5) & 0 \\ 0 & 0 & 6\alpha_7 \end{bmatrix}$$

- This then results in three unknown parameters α_4 - α_6 that need to be derived from TXC investigations at different fabric angles.



- We define the peak deviatoric stress values in terms of the angle of lamination for a constant confinement pressure;
- We solve for the matrix of unknown parameters using an incremental (iterative) solution of the deviatoric stress equation to come up with the projection matrix \mathbf{P}_{orth}

$$q_{orth} = \sqrt{\frac{1}{2}(\sigma^l)^T \mathbf{P}_{orth} \sigma^l}$$





Porous Flow - Orthotropy

■ Orthotropy – Flow/Heat

- We employ a transformation of the global coordinate system stress/strain variables to the material lamination aligned system;
- We employ an analogous transformation approach for the material properties required for the porous flow solution;
- Standard isotropic porous flow modelling requires specification of:
 - The isotropic permeability k_{iso} ;
 - The material porosity (voids ratio) & saturation;
 - The grain and fluid stiffness's;
 - The Biot constant;

- We are then able to define the orthotropic permeability in the transformed (lamination specific) coordinate system with simple scalar multiplication of the isotropic intrinsic permeability, such that

$$k_{ortho} = \begin{bmatrix} f1.k_{iso} & 0 & 0 \\ 0 & f2.k_{iso} & 0 \\ 0 & 0 & f3.k_{iso} \end{bmatrix}$$

- Where the constants f1-f3 are the factors defining the ratio to the isotropic permeability.
- The coordinate system transformation employed for the porous flow properties is identical to that for both the mechanical (strength) and thermal field types.



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